AP Calculus BC
Chapter 7 – AP Exam Problems

Position – Velocity – Acceleration

1. The velocity of a particle moving on a line at time t is $v(t) = 3t^{1/2} + 5t^{3/2}$ meters per second. How many meters did the particle travel from t = 0 to t = 4?

   A) 32 B) 40 C) 64 D) 80 E) 184

2. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from t = 0 to t = 2?

   A) $e^2 - 1$ B) $e - 1$ C) $2e$ D) e^2 E) $\frac{e^3}{3}$

3. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when t = 3 and the position is 10 when t = 1, then the position $x(t) =$

   A) $9t^2 + 1$
   B) $3t^2 - 2t + 4$
   C) $t^3 - t^2 + 4t + 6$
   D) $t^3 - t^2 + 9t - 20$
   E) $36t^3 - 4t^2 - 77t + 55$

4. If the velocity of a particle moving along the x-axis is $v(t) = 2t - 4$ and if at t = 0 its position is 4, then at any time t its position $x(t)$ is

   A) $t^2 - t$
   B) $t^2 - 4t - 4$
   C) $t^2 - 4t + 4$
   D) $2t^2 - 4t$
   E) $2t^2 - 4t + 4$

5. A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far (in meters) does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

   A) 20 B) 14 C) 7 D) 6 E) 3

6. A particle moves along the x-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at t = 0 the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time t > 0 is $x(t) =$

   A) $-\frac{e^{-2t}}{2} + 3$
   B) $\frac{e^{-2t}}{4} + 4$
   C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$
   D) $\frac{e^{-2t}}{4} + 3t + \frac{15}{4}$
   E) $\frac{e^{-2t}}{4} + 3t + 4$

7. (1985 BC1) A particle moves along the x-axis with acceleration given by $a(t) = \cos t$ for $t \geq 0$. At $t = 0$ the velocity $v(t)$ of the particle is 2 and the position $x(t)$ is 5.

   (a) Write an expression for the velocity $v(t)$ of the particle.
   (b) Write an expression for the position $x(t)$.
   (c) For what values of t is the particle moving to the right? Justify your answer.
   (d) Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.

8. (1986 BC1) A particle moves along the x-axis so that, at any time $t \geq 1$, its acceleration is given by $a(t) = \frac{1}{t}$. At time $t = 1$, the velocity of the particle is -2 and its position is 4.

   (a) Find the velocity $v(t)$ for $t \geq 1$.
   (b) Find the position $x(t)$ for $t \geq 1$.
   (c) What is the position of the particle when it is farthest to the left?

9. (1990 AB1) A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

   (a) Find the values of t for which the particle is at rest.
   (b) Write an expression for the position $x(t)$ of the particle at any time $t \geq 0$.
   (c) Find the total distance traveled by the particle from $t = 0$ to t = 2.

10. (1991 BC1) A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

   (a) Find the position $x(t)$ of the particle at any time $t \geq 0$.
   (b) Find all values of t for which the particle is at rest.
   (c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.
   (d) Find the total distance traveled by the particle from $t = 0$ to t = 2.

11. (1994 AB4) A particle moves along the x-axis so that its velocity at any time $t > 0$ is given by $v(t) = t \ln t - t$. At time $t = 1$, the position of the particle is $x(1) = 6$.

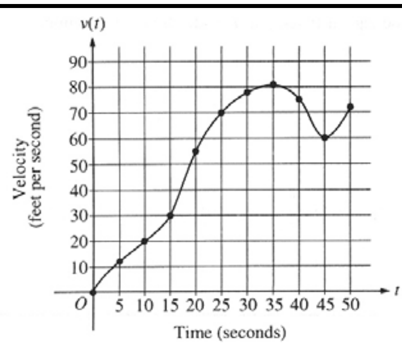
   (a) Write an expression for the acceleration of the particle.
   (b) For what values of t is the particle moving to the right?
   (c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.
   (d) Write an expression for the position $x(t)$ of the particle.

12. (1995 AB2) A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t\cos t$. At $t = 0$, the position of the particle is $y = 3$.

(a) For what values of $t, 0 \leq t \leq 5$, is the particle moving upward?
(b) Write an expression for the acceleration of the particle in terms of t.
(c) Write an expression for the position $y(t)$ of the particle.
(d) For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

13. (1997 AB1) A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.
(b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0, 3]?
(c) Find the total distance traveled by the particle from time $t = 0$ to time $t = 3$.



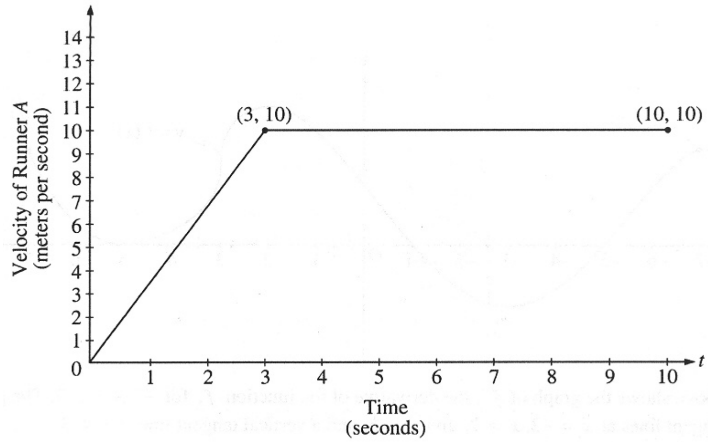
t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

14. (1998 AB3) The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5-second time intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.
(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.
(d) Approximate $\int_0^{50} v(t)dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

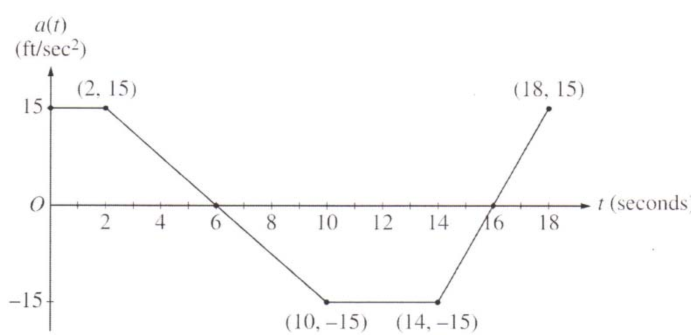
15. (1999 AB1) A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t\sin\left(t^2\right)$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
(d) Find the total distance traveled by the particle from $t = 0$ to t = 2.



16. (2000 BC2) Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

    (a) Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure.
    (b) Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.
    (c) Find the total distance of Runner A and the total distance of Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.



17. (2001 BC3) A car is traveling on a straight road with velocity 55 ft/sec at time t = 0. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

    (a) Is the velocity of the car increasing at t = 2 seconds? Why or why not?
    (b) At what time in the interval $0 \le t \le 18$, other than t = 0, is the velocity of the car 55 ft/sec? Why?
    (c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
    (d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

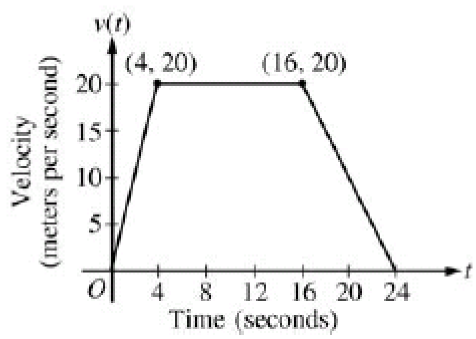
18. (2003 AB2) A particle moves along the x-axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$. At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?
(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.
(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.
(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

19. (2004 AB3) A particle moves along the y-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x$ = arctan x)

(a) Find the acceleration of the particle at time $t = 2$.
(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
(c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.
(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

20. (2005 BC5) A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph below.



(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.
(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

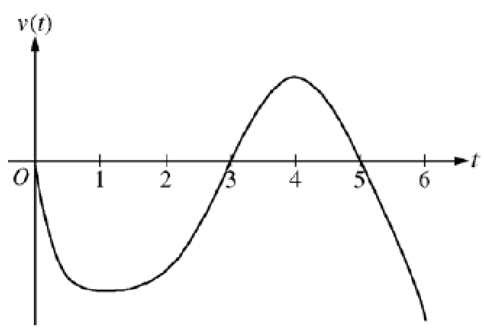
t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

21. (2006 BC4) Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

   (a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)dt$.

   (c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.



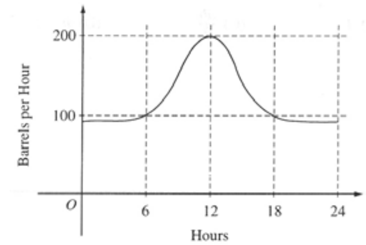
Graph of v

22. (2008 BC4) A particle moves along the x – axis so that its velocity at time t, for $0 \le t \le 6$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t – axis and the graph of v on the intervals [0, 3], [3, 5], and [5, 6] are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

   (a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
   (b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.
   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
   (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

Antiderivatives as Amounts

23. The oil flow, in barrels per hour, through a pipeline on July 9 is given by the graph. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?



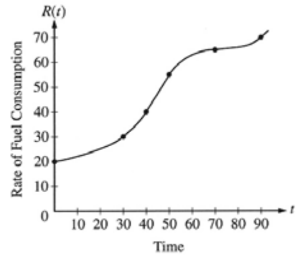
A) 500 B) 600 C) 2,400 D) 3,000 E) 4,800

24. (2004 BC1) Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$ for $0 \le t \le 30$, where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30 minute period?
(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.
(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

25. (2002 BC2) The rate at which people enter an amusement park on a given day is modeled by $E(t) = \frac{15600}{t^2 - 24t + 160}$. The rate at which people leave the same amusement park on the same day is modeled by the function L defined by $L(t) = \frac{9890}{t^2 - 38t + 370}$. Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.
(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
(c) Let $H(t) = \int_9^t \left(E(x) - L(x)\right)dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.
(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

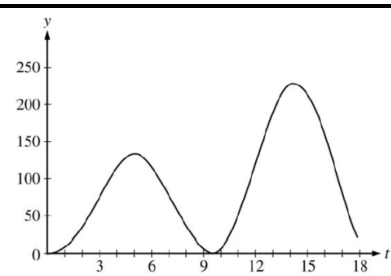
26. (2003 AB3) The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)dt$ using a left Riemann sum with five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

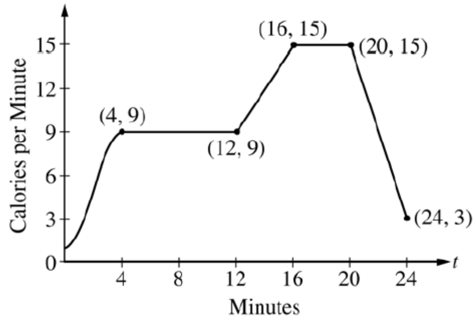


27. (2006 BC2) At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restriction when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.



28. (2006B BC4) The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function *f*. In the figure, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and *f* is piecewise linear for $4 \le t \le 24$.

    (a) Find $f'(22)$. Indicate units of measure.
    (b) For the time interval $0 \le t \le 24$, at what time *t* is *f* increasing at its greatest rate? Show the reasoning that supports your answer.
    (c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.
    (d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find *c* so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

29. (2008 BC2) Concert tickets went on sale at noon (t = 0) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time *t* is modeled by a twice-differentiable function *L* for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

    (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. (t = 5.5). Show the computations that lead to your answer. Indicate units of measure.
    (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
    (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
    (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. (t = 3), to the nearest whole number?

30. (2008B BC2) For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?
(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.
(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

Areas

31. The area of the region enclosed by the curve $y = \dfrac{1}{x-1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

A) $\dfrac{5}{36}$ B) $\ln\dfrac{2}{3}$ C) $\ln\dfrac{4}{3}$ D) $\ln\dfrac{3}{2}$ E) $\ln 6$

32. If $0 \leq k < \dfrac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \dfrac{\pi}{2}$ is 0.1, then k =

A) 1.471 B) 1.414 C) 1.277 D) 1.120 E) 0.436

33. The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

A) 36 B) 23 C) 20 D) 17 E) 9

34. The area of the region in the first quadrant enclosed by the graph of $y = x(1-x)$ and the x-axis is

A) $\dfrac{1}{6}$ B) $\dfrac{1}{3}$ C) $\dfrac{2}{3}$ D) $\dfrac{5}{6}$ E) 1

35. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

A) $\dfrac{1}{6}$ B) $\dfrac{1}{3}$ C) $\dfrac{1}{2}$ D) $\dfrac{5}{6}$ E) 1

36. Let R be the region enclosed by the graph of $y = 1 + \ln\left(\cos^4 x\right)$, the x-axis, and the lines $x = -\dfrac{2}{3}$ and $x = \dfrac{2}{3}$. The closest integer approximation of the area of R is

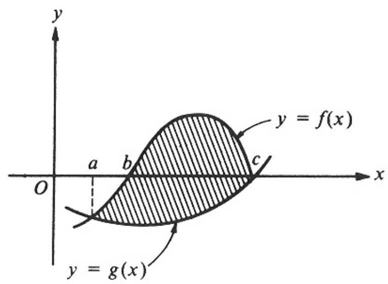
A) 0 B) 1 C) 2 D) 3 E) 4

37. The area of the region in the <u>first quadrant</u> that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) 1 E) $\frac{65}{4}$

38. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

A) $\frac{2}{3}$ B) 1 C) $\frac{4}{3}$ D) 2 E) $\frac{14}{3}$



39. The area of the shaded region in the figure above is represented by which of the following integrals?

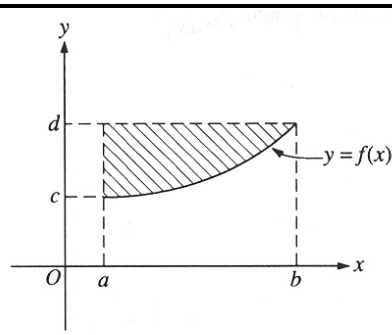
A) $\int_a^c \left(|f(x)| - |g(x)|\right)dx$

B) $\int_b^c f(x)dx - \int_a^c g(x)dx$

C) $\int_a^c \left(g(x) - f(x)\right)dx$

D) $\int_a^c \left(f(x) - g(x)\right)dx$

E) $\int_a^b \left(g(x) - f(x)\right)dx + \int_b^c \left(f(x) - g(x)\right)dx$



40. Which of the following represents the area of the shaded region in the figure above?

A) $\int_c^d f(y)dy$

B) $\int_a^b \left(d - f(x)\right)dx$

C) $f'(b) - f'(a)$

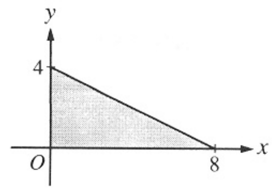
D) $(b-a)[f(b) - f(a)]$

E) $(d-c)[f(b) - f(a)]$

41. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

A) $\frac{2}{3}$ B) $\frac{8}{3}$ C) 4 D) $\frac{14}{3}$ E) $\frac{16}{3}$

Volume



42. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

A) 12.566 B) 14.661 C) 16.755 D) 67.021 E) 134.041

43. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

A) $\frac{1-e^{-6}}{2}$ B) $\frac{1}{2}e^{-6}$ C) e^{-6} D) e^{-3} E) $1-e^{-3}$

44. The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

A) $\frac{4\pi}{3}$ B) $\frac{16\pi}{5}$ C) $\frac{4}{3}$ D) $\frac{16}{5}$ E) $\frac{64}{5}$

45. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

A) 2π B) 4π C) 6π D) 9π E) 12π

46. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

A) 3π B) $2\sqrt{3}\pi$ C) $\frac{9}{2}\pi$ D) 9π E) $\frac{36\sqrt{3}}{5}\pi$

47. The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when R is revolved about the <u>y-axis</u> is

A) 6π B) 8π C) $\frac{34\pi}{3}$ D) 16π E) $\frac{544\pi}{15}$

48. What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \dfrac{\pi}{3}$.

A) $\dfrac{\pi}{\sqrt{3}}$ B) π C) $\pi\sqrt{3}$ D) $\dfrac{8\pi}{3}$ E) $\pi \ln\left(\dfrac{1}{2}+\sqrt{3}\right)$

49. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \dfrac{\pi}{2}$. The volume of the solid obtained by revolving R about the x-axis is given by

A) $2\pi\int_0^{\pi/2} x\sin x\,dx$ C) $\pi\int_0^{\pi/2} (1-\sin x)^2\,dx$ E) $\pi\int_0^{\pi/2} (1-\sin^2 x)\,dx$

B) $2\pi\int_0^{\pi/2} x\cos x\,dx$ D) $\pi\int_0^{\pi/2} \sin^2 x\,dx$

50. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

A) 8π B) $\dfrac{32\pi}{5}$ C) $\dfrac{16\pi}{3}$ D) 4π E) $\dfrac{8\pi}{3}$

51. A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

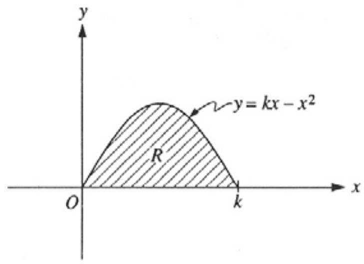
A) $2\pi\int_0^1 xe^{2x}\,dx$ C) $\pi\int_0^1 e^{4x}\,dx$ E) $\dfrac{\pi}{4}\int_0^e \ln^2 y\,dy$

B) $2\pi\int_0^1 e^{2x}\,dx$ D) $\pi\int_0^e y\ln y\,dy$

52. The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

A) $\int_0^6 \pi\left(6x-x^2\right)^2 dx$ C) $\int_0^6 \pi x\left(6x-x^2\right)^2 dx$ E) $\int_0^9 \pi\left(3+\sqrt{9-y}\right)^2 dy$

B) $\int_0^6 2\pi x\left(6x-x^2\right)dx$ D) $\int_0^6 \pi\left(3+\sqrt{9-y}\right)^2 dy$



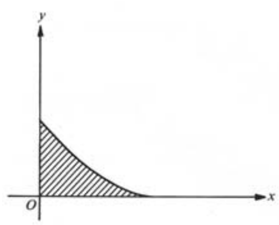
53. The shaded region R, shown in the figure above, is rotated about the y-axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates k?

A) 1.51 B) 2.09 C) 2.49 D) 4.18 E) 4.77

Area and Volume FRQ

54. (1987 BC3) Let R be the region enclosed by the graph of $y = \ln x$, the line $x = 3$, and the x-axis.

(a) Find the area of region R.
(b) Find the volume of the solid generated when R is revolved about the x-axis.
(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid formed by revolving R about the line $x = 3$.

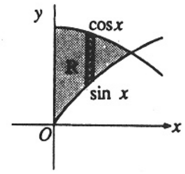


55. (1988 BC5) The base of a solid S is the shaded region in the xy-plane enclosed by the x-axis, the y-axis, and the graph of $y = 1 - \sin x$, as shown in the figure above. For each x, the cross section of S perpendicular to the x-axis at the point $(x, 0)$ is an isosceles right triangle whose hypotenuse lies in the xy-plane.

(a) Find the area of the triangle as a function of x.
(b) Find the volume of S.

56. (1990 BC2) Let R be the region in the xy-plane between the graphs of $y = e^x$ and $y = e^{-x}$ from $x = 0$ to $x = 2$.

(a) Find the volume of the solid generated when R is revolved about the x-axis.
(b) Find the volume of the solid generated when R is revolved about the y-axis.



57. (1991 BC3) Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = \sin x$ and $y = \cos x$, as shown in the figure above.

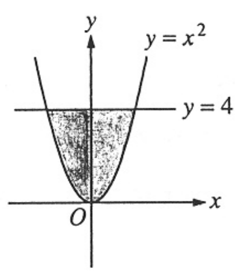
(a) Find the area of region R.
(b) Find the volume of the solid generated when R is revolved about the <u>x-axis</u>.
(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.

58. (1998 AB1) Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

(a) Find the area of the region R.
(b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
(c) Find the volume of the solid generated when R is revolved about the x-axis.
(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

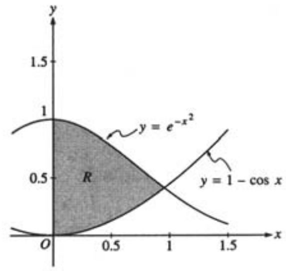
59. (1998 BC1) Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the x-axis, and the y-axis.

(a) Find the area of the region R.
(b) Find the volume of the solid generated when R is revolved about the x-axis.
(c) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.



60. (1999 BC2) The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.
(b) Find the volume of the solid generated when R is revolved about the <u>x-axis</u>.
(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

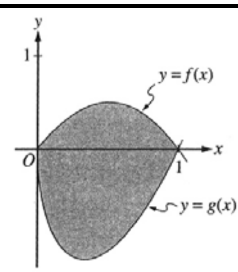


61. (2000 BC1) Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of region R.
(b) Find the volume of the solid generated when R is revolved about the x-axis.
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

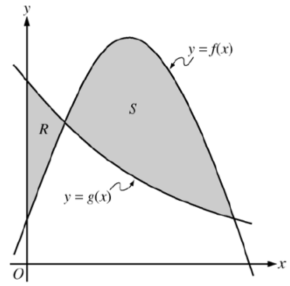
62. (2002 BC1) Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region between f and g between $x = \frac{1}{2}$ and $x = 1$.
(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.
(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$.



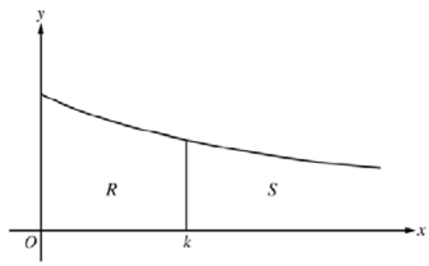
63. (2004 BC2) Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.
(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.
(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.



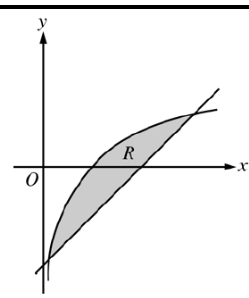
64. (2005 BC1) Let f and g be the functions given by $f(x)=\frac{1}{4}+\sin(\pi x)$ and $g(x)=4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y – axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure.

(a) Find the area of R.
(b) Find the area of S.
(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.



65. (2005B BC6) Consider the graph of the function f given by $f(x)=\frac{1}{x+2}$ for $x \geq 0$, as shown in the figure above. Let R be the region bounded by the graph of f, the x – and y – axes, and the vertical line $x = k$, where $k \geq 0$.

(a) Find the area of R in terms of k.
(b) Find the volume of the solid generated when R is revolved about the x – axis in terms of k.



66. (2006 BC1) Let R be the shaded region bounded by the graph of $y=\ln x$ and the line $y=x-2$, as shown above.

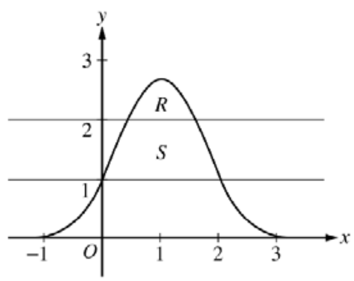
(a) Find the area of R.
(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.
(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y – axis.

67. (2006B BC1) Let f be the function given by $f(x)=\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and the line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the area of R.
(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

68. (2007 BC1) Let R be the region in the first and second quadrants bounded above by the graph of $y=\frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.
(b) Find the volume of the solid generated when R is rotated about the x – axis.
(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x – axis are semicircles. Find the volume of this solid.



69. (2007B BC1) Let R be the region bounded by the graph of $y=e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y=e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.
(b) Find the area of S.
(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

Arc Length

70. What is the length of the arc of $y=\frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 3$?

A) $\frac{8}{3}$ B) 4 C) $\frac{14}{3}$ D) $\frac{16}{3}$ E) 7

71. The length of the curve $y=x^3$ from $x = 0$ to $x = 2$ is given by

A) $\int_0^2 \sqrt{1+x^6}\,dx$
B) $\int_0^2 \sqrt{1+3x^2}\,dx$
C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$
D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$
E) $\int_0^2 \sqrt{1+9x^4}\,dx$

Multiple Choice Answers

Position-Velocity-Acceleration

1.	D	1985	AB14	74%
2.	A	1988	AB3	83%
3.	C	1993	AB11	86%
4.	C	1985	BC15	92%
5.	B	1988	BC12	64%
6.	E	1993	BC20	83%

Antiderivatives as Amounts

23.	D	1998	AB9	67%

Areas in the Plane

31.	D	1993	AB6	70%
32.	D	1998	AB92	76%
33.	D	1985	BC1	94%
34.	A	1988	BC1	90%
35.	A	1993	BC1	94%
36.	B	1998	BC80	91%
37.	A	1985	AB34	59%
38.	C	1988	AB21	63%
39.	D	1988	AB34	50%
40.	B	1993	AB2	87%
41.	D	1998	AB25	71%

Volumes

42.	C	1998	AB86	19%
43.	A	1985	BC39	51%
44.	D	1988	BC25	56%
45.	B	1988	AB43	24%
46.	C	1993	AB30	58%
47.	B	1988	BC29	56%
48.	C	1993	BC30	70%
49.	E	1988	BC36	48%
50.	A	1985	AB45	51%
51.	A	1988	AB30	57%
52.	B	1985	BC35	70%
53.	B	1993	BC19	59%

Arc Length

70.	C	1985	BC41	71%
71.	E	1988	BC33	84%